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# Diffusion in porous media – measurement and modeling

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#### Diffusion – what is that?



### Diffusion – what is that?





From G. Gamow "One, Two, Three...Infinity" The Viking Press, New York, 1955.

Dresden, Frauenkirche 2010

#### Diffusion – application to societies



Fig. 5: Advance of agriculture from the Near East and Anatolia over a distance of 4.000 km via Greece and the Balkans to Central and finally Western and Northern Europe [after 18], dates BC according to present state of knowledge, courtesy of E. Lenneis.

in:

Diffusion and Brownian Motion Analogies in the Migration of Atoms, Animals, Men and Ideas

#### Diffusion – application to societies

#### diffusion-fundamentals

The Open-Access Journal for the Basic Principles of Diffusion Theory, Experiment and Application

Diffusion and Brownian Motion Analogies in the Migration of Atoms, Animals, Men and Ideas

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#### Abstract

The macroscopic laws of diffusion were laid down for the case of liquids by Adolf Fick 150 years ago who realised the analogy of diffusion and heat conduction. 100 years ago Einstein and Smoluchowski put up the equation named after these scientists teaching us how to trace down the motion of a single diffusing particle and thus to understand long time unexplained Brownian motion as a fluctuation phenomenon. In the last fifty years these laws and their combination were boldly but successfully applied to the diffusion, migration, dispersion of single atoms, men, animals and ideas.

We start by showing how the Einstein-Smoluchowski equation makes possible to induce diffusivity from microscopic information on details of the diffusion jump in solids. We then report on Brownian motion and diffusion of our fore-fathers in the Neolithicum following Cavalli-Sforza's ideas and show how this diffusion must have been a mixture of demic diffusion, i.e. the diffusion of people, and the diffusion of technological ideas. Next we risk a glimpse to the immigration of early Americans. We point out the discrepancy a physicist faces in the conclusions of the Archaeologists. We finally discuss the ultra-fast dispersion of the horse-chestnut leaf miner throughout Europe following recent work of ecologists.

### Diffusion – first experiment

#### **Robert Brown**



#### **Robert Brown**, (1773-1958)

"Scottish botanist best known for his description of the natural continuous <u>motion</u> of minute particles in solution, which came to be called <u>Brownian movement</u>."

http://www.britannica.com/EBchecked/topic/81618/Robert-Brown

### Diffusion – first experiment

#### **Robert Brown**



"In 1827, while examining grains of pollen of the plant <u>Clarkia pulchella</u> suspended in water under a <u>microscope</u>, Brown observed minute particles, now known to be amyloplasts (starch organelles) and spherosomes (lipid organelles), ejected from the pollen grains, executing a continuous jittery motion. He then observed the same motion in particles of inorganic matter, enabling him to rule out the hypothesis that the effect was liferelated....."

http://en.wikipedia.org/wiki/Robert\_Brown\_(botanist)

http://www.uni-leipzig.de/diffusion/pdf/volume4/diff\_fund\_4(2006)6.pdf

#### diffusion-fundamentals

The Open-Access Journal for the Basic Principles of Diffusion Theory, Experiment and Application

#### One and a Half Century of Diffusion: Fick, Einstein, Before and Beyond

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Extended version of Jean Philibert's contribution to the Proceedings of Diffusion Fundamentals I, Leipzig, 2005

#### Abstract

The year 2005 gave us, through two anniversaries (1855 Fick and 1905 Einstein), the wish to go back to these authors' seminal papers, whose aftermath had been (and still is) prodigious. This essay describes the contents of these articles: the macroscopic approach with Fick equations and the microscopic one with the Einstein-Smoluchowski random walk (Brownian motion) equation, while considering them in their historical context. Some further developments are briefly discussed.

#### A. E. Fick



"Adolf Eugen Fick (1829-1901) was a <u>German physiologist</u>. He started to study mathematics and physics, but then realized he was more interested in medicine. He earned his <u>doctorate</u> in medicine at <u>Marburg</u> in 1851."

"In 1855 he introduced Fick's law of diffusion, which governs the diffusion of a gas across a fluid membrane."

http://en.wikipedia.org/wiki/Adolf\_Fick

#### Albert Einstein (1879-1955)



Marian von Smoluchowski (1872-1917 was a phyiscist.



http://de.wikipedia.org/wiki/Albert\_Einstein http://de.wikipedia.org/wiki/Marian\_Smoluchowski

#### A. Einstein, M.Smoluchowski

In <u>physics</u> (namely, in <u>kinetic theory</u>) the **Einstein relation** (also known as **Einstein– relation**) is a previously unexpected connection revealed independently by <u>Albert Einstein</u> in 1905 and by <u>Marian Smoluchowski</u> (1906) in their papers on <u>Brownian motion</u>.

Two important special cases of the relation are:

(diffusion of <u>charged</u> particles)

$$D = \frac{\mu_q \, \kappa_B r}{q}$$
$$D = \frac{k_B T}{6\pi \, \eta \, r}$$

 $\mu h_{-}T$ 

("**Einstein–Stokes equation**", for diffusion of spherical particles through liquid with low <u>Reynolds number</u>)

$$D = \mu \, k_B T^D = \mu \, k_B T$$

The more general form of the equation is:

where the "mobility"  $\mu$  is the ratio of the particle's terminal <u>drift velocity</u> to an applied <u>force</u>,  $\mu = v_d / F$ .

This equation is an early example of a <u>fluctuation-dissipation relation</u>. It is frequently used in the electrodiffusion phenomena.

http://en.wikipedia.org/wiki/Einstein\_relation\_(kinetic\_theory)

#### Importance of transport for catalysis

- Material and energy balances are required for both the fluid, which occupies the interstitial region between catalyst particles, and the catalyst particles, in which the reactions occur
- Determination of *rate limiting steps* to ensure which step acts to influence the overall rate of reaction in the pellet



Interaction of chemical reaction and transport processes



#### What?

- □ Introduction to porosity, diffusion
- Overview to methods for diffusion in porous (and other) systems
- General aspects of pulse methods
- □ Frequency Response approach
- Temporal Analysis of Products
- Literature recommendations

# Porosity

#### Porous materials in nature and industry

- porous materials: sand stone, porous rock, filter paper, nano tubes...
- main feature: *cavities* in a *solid matrix*
- cavities can be partly or fully connected
- accessible for probe molecules



 porosities are often desired and of importance in medicine, membranes, sorbents, ceramics, and *catalysts*





- pressing of support material leads to a network of micro-/meso- and macroporous areas: powder particles are micro-/mesoporous, pores between particles build up a macroporous network
- types of pores
  - open pores: surface ~, column ~, hollow ~
  - isolated pores: inclusion ~
  - see IUPAC



- Porous solid: a solid with pores, i.e. cavities, channels or interstices, which are deeper than they are wide
- Pore size: (generally pore width): the distance between two opposite walls of the pore
  - Micropores (< 2 nm),
  - Mesopores (2-50 nm)
  - Macropores (> 50 nm)
- Porosity 
   *ɛ*: ratio of the total pore volume V<sub>P</sub> to the apparent volume V of the particle or powder (excluding interparticle voids)

### Types of porosity

- bed porosity and particle porosity have to be differentiated !
- bed porosity (or void fraction) *E*<sub>B</sub> is defined as the volume of voids per volume of reactor
- *porosity* (or pellet void fraction) is defined as  $\varepsilon = \rho_P * V_g$ ( $\rho_P$  effective particle density,  $V_g$  pore volume)

Tabelle 10.1: Dichte und Porositäten von Adsorbentien.

$\varrho$ :	$\varrho$ : Dichte; $\varrho_{\rm sch}$ : Schüttdichte; $\epsilon$ : Porosität der Schüttung; $\epsilon_0$ : Poro	sität der	Adsorbens;
<i>s</i> :	s: spezifische Oberfläche; $\overline{d}_{kp}$ : mittlerer Porendurchmesser; vgl. a	uch Tab.	5.4.

Adsorbens	$\varrho/\mathrm{kgm^{-3}}$	$ ho_{\rm sch}/{\rm kgm^{-3}}$	$\epsilon/1$	$\epsilon_0/1$	$s/\mathrm{m}^2\mathrm{g}^{-1}$	$\overline{d}_{\mathrm{kp}}/\mathrm{nm}$
Aktivkohle Aktivkohle eng aktive Tonerde Kieselgel eng Kieselgel weit Zeolithe	$2200 \\ 2000 \\ 3000 \\ 2200 \\ 2200 \\ 2600$	$600\\800\\1200\\1100\\1100\\1300$	0.73 0.60 0.60 0.50 0.50 0.50	$\begin{array}{c} 0.33 \\ 0.44 \\ 0.38 \\ 0.32 \\ 0.45 \\ 0.42 \end{array}$	$     1200 \\     800 \\     300 \\     700 \\     300 \\     700    $	20 33 20 100 3 bis 10

#### Jakubith

## Diffusion – Background

#### Diffusion – in general ...

- ...is the *transport* of mass in gases, liquids and solids under the influence of a *concentration gradient*
- ...proceeds spontaneously due to microscopic movement of mass
- ... is an *irreversible process* which leads to an increase in entropy and is only reversible by supply of work (see textbooks of thermodynamics)

#### Diffusion – in special ...

- ...*mechanisms differ* for gases/liquids and solids
- ...for gases/liquids: statistical movement according to T
- ...for solids: different mechanisms possible:
  - i) direct exchange of lattice places
  - ii) movement via interstitial lattice sites
  - iii) movement via lattice vacancies
  - iv) movement via lattice defects or on the grain surface
  - v) exchange of sites on the crystal surface

#### Examples - Diffusion in technical solids

#### • High temperature inorganic reactions

Reaktion/Verfahren	Temperatur in °C	Reaktor
Herstellung von Gläsern	700 1900	direkt beheizte Schmelz- wannen
Herstellung von Emails	1000 1300	kurzer Drehrohrofen
Herstellung von keramischen Materialien	890 1900	Tunnel-, Rund-, Ring-, Pendelhaubenofen
Herstellung von Zement	1100 1450	Drehrohrofen
Thermische Zersetzung von Calciumcarbonat	800 1300	Schachtofen
Calcinierung von Al(OH) <sub>3</sub> zu Al <sub>2</sub> O <sub>3</sub>	1200	Drehrohrofen
Thermische Metallgewinnung durch Zersetzung von Metallhalogeniden (Zr, Ti) nach dem Aufwachs- verfahren (van-Arkel-Prozeß)	500 1500	elektrischer Ofen
Herstellung von Chromaten aus Eisen-Chromium-Oxiden	900 1200	Drehrohrofen, Ringherdofen
Thermische Spaltung von Metall- chloriden zu Oxiden (MgO, $Al_2O_3$ , $Fe_2O_3$ )	600 1200	Sprühreaktor (Spaltrohr)
Gewinnung von SO <sub>2</sub> aus Calcium- sulfat	1100 1400	Drehrohrofen

Tabelle 2.29. Technisch wichtige anorganische Hochtemperaturreaktionen

#### TC Anorg. Chemie

### **Diffusion - quantitatively**

- ...generally *increases* with temperature
- ...generally decreases with increasing density
- ...the equilibration takes
  - minutes for gases
  - days/weeks for liquids
  - with measurable rate only close to the melting point for solids
- ... characteristic quantity: *diffusion coefficient*
- ...can be described as *molecular transport quantity* or as *effective quantity*

### Examples - Diffusion coefficients

	D [cm <sup>2</sup> /s]	T [K]
$H_2$ in $O_2$	0.78	298
$N_2$ in $O_2$	0.19	298
H <sub>2</sub> O vapour in air	0.23	298
CH <sub>3</sub> OH in water	<b>1.2*10</b> <sup>-5</sup>	298
Sugar in water	0.5*10 <sup>-5</sup>	298
Au in Cu	2.1*10 <sup>-11</sup>	1023
Fe in Al	6.2*10 <sup>-14</sup>	623
<sup>36</sup> Cl <sup>-</sup> in AgCl	<b>3.2*10</b> <sup>-16</sup>	623

Lexikon Chemie

l'able 17.1-1	Experimental Diffusivities <sup>a</sup> and Limiting Schmidt
Numbers <sup>b</sup> of	Gas Pairs at 1 Atmosphere Pressure

Gas pair	Temperature	DAR	Sc		
A-B	(K)	$(cm^2/s)$	$x_A \rightarrow 1$	$x_B \rightarrow 1$	
CO <sub>2</sub> -N <sub>2</sub> O	273.2	0.096	0.73	0.72	
CO <sub>2</sub> -CO	273.2	0.139	0.50	0.96	
CO <sub>2</sub> -N <sub>2</sub>	273.2	0.144	0.48	0.91	
	288.2	0.158	0.49	0.92	
	298.2	0.165	0.50	0.93	
$N_2 - C_2 H_6$	298.2	0.148	1.04	0.50	
$N_2 - nC_4 H_{10}$	298.2	0.0960	1.60	0.33	
N <sub>2</sub> -O <sub>2</sub>	273.2	0.181	0.72	0.55	
H <sub>2</sub> -SF <sub>6</sub>	298.2	0.420	3 37	0.74	
H <sub>2</sub> CH <sub>4</sub>	298.2	0.726	1.95	0.055	
$H_2 - N_2$	273.2	0.674	1.95	0.25	
NH <sub>3</sub> -H <sub>2</sub> <sup>c</sup>	263	0.58	0.10	0.19	
NH <sub>3</sub> -N <sub>2</sub> <sup>c</sup>	298	0.233	0.19	1.53	
H <sub>2</sub> O-N <sub>2</sub> <sup>c</sup>	308	0.259	0.62	0.65	
H <sub>2</sub> O-O <sub>2</sub> <sup>c</sup>	352	0.259	0.56	0.62	
$C_{3}H_{8}-nC_{4}H_{10}^{d}$	378.2	0.0768	0.56	0.59	
	437.7	0.107	0.95	0.66	
$C_{3}H_{8}-iC_{4}H_{10}^{d}$	298.0	0.0430	0.91	0.63	
	378.2	0.0823	1.04	0.73	
	437.8	0.112	0.09	0.63	
C <sub>3</sub> H <sub>8</sub> -neo-C <sub>5</sub> H <sub>12</sub> <sup>d</sup>	298.1	0.0431	1.07	0.61	
	378.2	0.0703	1.06	0.56	
	437.7	0.0945	1.04	0.55	
$nC_4H_{10}$ -neo- $C_5H_{12}^d$	298.0	0.0343	1.03	0.55	
	378.2	0.0413	0.76	0.59	
	437.8	0.0044	0.78	0.61	
iC4H10-neo-C5H12d	298.1	0.0359	0.80	0.62	
5 -12	378.2	0.0562	0.89	0.67	
	437.7	0.0380	0.89	0.67	
		0.0786	0.87	0.66	

<sup>a</sup> Unless otherwise indicated, the values are taken from J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, *Molecular Theory of Gases and Liquids*, 2nd corrected printing, Wiley, New York (1964), p. 579. All values are given for 1 atmosphere pressure.

<sup>b</sup> Calculated using the Lennard-Jones parameters of Table E.1. The parameters for sulfur hexafluoride were obtained from second virial coefficient data.

<sup>c</sup> Values of  $\mathfrak{D}_{AB}$  for the water and ammonia mixtures are taken from the tabulation of R. C. Reid, J. M. Prausnitz, and B. E. Poling, *The Properties of Gases and Liquids*, 4th edition, McGraw-Hill, New York (1987).

<sup>*d*</sup> Values of  $\mathfrak{D}_{AB}$  for the hydrocarbon–hydrocarbon pairs are taken from S. Gotoh, M. Manner, J. P. Sørensen, and W. E. Stewart, *J. Chem. Eng. Data*, **19**, 169–171 (1974).

<sup>e</sup> Values of μ for water and ammonia were calculated from functions provided by T. E. Daubert, R. P. Danner, H. M. Sibul, C. C. Stebbins, J. L. Oscarson, R. L. Rowley, W. V. Wilding, M. E. Adams, T. L. Marshall, and N. A. Zundel, *DIPPR*<sup>®</sup>, *Data Compilation of Pure Compound Properties*, Design Institute for Physical Property Data<sup>®</sup>, AIChE, New York, N.Y. (2000). **Table 17.1-4** Experimental Diffusivities of Gases in Polymers.<sup>*a*</sup> Diffusivities,  $\mathfrak{D}_{AB}$ , are given in units of  $10^{-6}$  (cm<sup>2</sup>/s). The values for N<sub>2</sub> and O<sub>2</sub> are for 298K, and those for CO<sub>2</sub> and H<sub>2</sub> are for 198K.

	N <sub>2</sub>	O <sub>2</sub>	CO <sub>2</sub>	H <sub>2</sub>
Polybutadiene	1.1	1.5	1.05	96
Silicone rubber	15	25	15	75
Trans-1,4-polyisoprene	0.50	0.70	0.47	5.0
Polystyrene	0.06	0.11	0.06	4.4

<sup>a</sup> Excerpted from D. W. van Krevelen, *Properties of Polymers*, 3rd edition, Elsevier, Amsterdam (1990), pp. 544–545. Another relevant reference is S. Pauly, in *Polymer Handbook*, 4th edition (J. Brandrup and E. H. Immergut, eds.), Wiley-Interscience, New York (1999), Chapter VI.

## Transport by Diffusion

#### General view to transport equations for gases



Eigenschaft	transportierte Größe	einfache kinetische Gastheorie	Einheiten
Diffusion	Materie	$D = \frac{1}{3}\lambda \bar{c}$	$m^2 s^{-1}$
Thermische Leitfähigkeit	Energie	$\kappa = \frac{1}{3}\lambda \bar{c}C_{V,m}[X]$ $= \frac{\bar{c}C_{V,m}}{(3\sqrt{2})\sigma N_{A}}$	$J \mathrm{K}^{-1} \mathrm{m}^{-1} \mathrm{s}^{-1}$
Viskosität	Impuls	$\eta = \frac{1}{3}\lambda \bar{c}m\mathcal{N}$ $= \frac{m\bar{c}}{(3\sqrt{2})\sigma}$	$kg m^{-1} s^{-1}$



Transport mechanisms in porous solids



Pore diffusion depending on pore diameter

# Diffusion - Special diffusion phenomena

- Molecular diffusion
- Knudsen diffusion
- Surface diffusion
  - lateral diffusion
  - not of technical importance
- Configurational diffusion
  - pore diameter within molecular dimensions (0.3-1 nm) as for zeolites
  - diffusion coefficients are smaller by some orders of magnitude

#### Molecular diffusion



Abb. 24.1 Bei der Diffusion vermischen sich die Moleküle einer Substanz mit den Molekülen einer anderen Substanz.



Abb. 24.7 Der Fluß von Teilchen gegen einen Konzentrationsgradienten. Das erste Ficksche Gesetz besagt, daß der Fluß der Materie (die Zahl der Teilchen pro Zeit- und Flächeneinheit) proportional zu dem Dichtegradienten am betreffenden Punkt ist.

#### Molecular diffusion

 mixture of two components A and B, concentration gradient (in one dimension y): under steady-state conditions the diffusional flow of one component is described by *1. Fick law*

$$J_{A} = -D_{AB}\left(\frac{dc_{A}}{dy}\right)$$

- D<sub>AB</sub> ...*binary molecular diffusion coefficient* of component A diffusing through B
- $D_{AB} = f($  molecular properties of A and B, T, c or p)

Ficks's Second Law



#### Molecular diffusion coefficient

• Molecular diffusion coefficient for gases

$$D_{M} = \frac{1}{3}v\lambda$$
; v...average velocity;  $\lambda$ ...mean free path length

$$D_{\rm M} = \frac{2}{3} \left(\frac{kT}{\pi m}\right)^{\frac{1}{2}} \frac{kT}{\pi d^2 p}$$

$$D_{M} \sim T^{\frac{3}{2}}, \sim \frac{1}{p}, \sim \frac{1}{m}, \sim \frac{1}{d^{2}}$$

 D<sub>M</sub> during catalytic process nearly constant; reaction rate constant changes exponentially !

#### Molecular diffusion coefficient

- Kinetic gas theory delivers only inaccurate values for transport coefficients
- Binary diffusion coefficients  $D_{AB}^{gs}$  for mixtures of gases (up to p  $\approx 2.5^{*}10^{6}$  Pa) (Hirschfelder/Curtiss/Bird)

$$D_{AB}^{gs} = 0.001834 \frac{\sqrt{T^3 \frac{M_{r,A} + M_{r,B}}{2M_{r,A} M_{r,B}}}}{p \sigma_{AB}^2 \Omega_D} , m^2 s^{-1}$$

Т	Temperature [K]
p	Pressure [bar]
$M_{r,A}$ , $M_{r,B}$	Relative molecular masses of molecules A and B
$\sigma^2_{\scriptscriptstyle AB}$	Average collision factor of gases A and B
$arOmega_{\scriptscriptstyle D}$	Collision integral with respect to force constants

### Knudsen diffusion

- under *low pressure conditions and/or* for *small pores*: collision of gas with pore wall > collision of gas with gas
- *mean free path length* of molecule > pore diameter

$$\Lambda = \frac{1}{\sqrt{2} \pi \sigma^2} \frac{V}{N_A}$$

- $\Lambda$  ...mean free path length
- $\sigma^2$ ...molecular cross-section
- V... gas molar volume at p
- N<sub>A</sub>/V at 298 K :  $c_{ges} \approx 3.10^{19}$  \*p (molecules/cm<sup>3</sup>) (dimension of p 10<sup>5</sup> Pa)
- mean free path length with typical  $\sigma$  (9-20\*10<sup>-16</sup> cm<sup>2</sup>)

$$\boxed{\Lambda \cong \frac{10^2}{p} \quad (nm)}$$
## Knudsen diffusion

$$\boxed{\Lambda \cong \frac{10^2}{p} \quad (nm)}$$

Conditions for Knudsen diffusion

d <sub>pore</sub> [nm]	<1000	<100	<10	<2
p [bar]	0.1	1	10	50

## Knudsen diffusion

Knudsen flow through one zylindrical pore

$$D_{K,i} = \frac{d_p}{3} \sqrt{\frac{8RT}{\pi M}} \approx 10^{-6} \frac{m^2}{s} (at \, 293 \, K, 0.01 \, MPa)$$

• for porous solids, the relative pore volume  $\epsilon_{\rm b}$  and the tortuousity factor  $\tau_{\rm K}$  have to be considered

$$D^{eff}_{K,i} = \frac{\varepsilon_P}{\tau_K} \frac{d_P}{3} \sqrt{\frac{8RT}{\pi M}}$$

effective diffusion coefficient

• estimation of  $\tau$  is a complex procedure (see literature)

# Effective diffusion coefficients

- diffusional flow in the pores may be described by an *effective diffusion coefficient*
- relation to surface: surface of pore mouths is representing only a part of the outer surface of a particle



- pores are not ideally cylindrical
- pores are connected by a network



# Diffusion in porous solids

- ✓ Characteristic value to characterize the influence of internal transport phenomena of heterogeneous reactions on the surface between a fluid phase and a porous solid is Da<sup>II</sup> (see textbooks)
- ✓  $Da^{II} = f(k, c_{external}, I_{characteristic}, D_{eff})$
- if fluid is gaseous: *diffusion* in pores *depends on* dimensions of *pore* system

- for heterogeneous reactions in a porous solid, the conditions of pressure or pore diameter may be such that the system is between Knudsen and molecular diffusion
- mean free path length ≈ pore diameter
- both equations for  $D_M$  and  $D_{Kn}$  apply

$$\boxed{\frac{1}{D^{*}} = \frac{1}{D_{M}} + \frac{1}{D_{Kn}}}$$

## Effective diffusivities

• Use of porosity and tortuosity factor

$$D_{eff} = \frac{\epsilon}{\tau} D$$

Catalyst	$\epsilon$	τ
100–110 $\mu$ m powder packed into a tube	0.416	1.56
pelletized $Cr_2O_3$ supported on $Al_2O_3$	0.22	2.5
pelletized boehmite alumina	0.34	2.7
Girdler G-58 Pd on alumina	0.39	2.8
Haldor-Topsøe MeOH synthesis catalyst	0.43	3.3
0.5% Pd on alumina	0.59	3.9
1.0% Pd on alumina	0.5	7.5
pelletized Ag/8.5% Ca alloy	0.3	6.0
pelletized Ag	0.3	10.0

Fixed-bed Catal.

# Diffusion – Measuring techniques

## Diffuison – measuring techniques



J. Kärger, University Leipzig Vasenkov & Kärger. *Magn. Res. Imag.* <u>23</u> (2005) 139.

## Diffusion – overview according to exper. conditions



## **PFG NMR**

#### **PFG NMR**

Pulsed field gradient NMR
(PGSE Pulsed gradient spin echo technique)

- PFG NMR ...measures directly the probability distribution of molecular displacements of the probe molecules within the sample, averaged over each individual starting point
- PFG NMR ... is sensitive to the mean molecular displacements within the sample in the direction of the field gradients applied, measured in the time interval between the field gradient pulses
- **PFG NMR** ... is a **non-invasive** technique
- PFG NMR ...time variation allows a distinction between individual and different ranges of molecular transport in complex systems



- PFG NMR...enables to measure diffusion in materials from 100 nm....100 μm → intracrystalline and long-range diffusion
- **PFG NMR...**has a **lower limit** for measurement of **D** ~ **10**<sup>-14</sup> **m**<sup>2</sup>/s

# PFG NMR and terms of diffusion

#### Transport Diffusion





(Fick's 1st law)



Self-diffusion by Tracer Exchange



$$j_x^* = -D\frac{\partial c^*}{\partial x}$$



Self-diffusion by following the individual molecules (QENS, PFG NMR)

$$\langle x^2(t) \rangle = 2Dt$$

**Einstein Equation** 





#### **QUENS** Quasi-elastic neutron scattering

- QUENS...is a sensitive probe for studying the structure and dynamics of materials on the atomic and molecular level
- QUENS...is working in the submicroscopic scale
- QUENS...studies the dynamics of the framework and of the adsorbed molecules
- QUENS...allows to follow the location of cations, water, adsorbed moelcules thus visualizing the elementary processes of diffusion
- QUENS...has a lower limit for measurement of D ~ 10<sup>-14</sup> m<sup>2</sup>/s
- QUENS...is applying mainly hydrogenated probe molecules (large neutron cross section of H<sub>2</sub>)

# IR imaging



# IR microscopy and IR imaging (IRM)

### IR Microscopy and IR Micro-Imaging (IRM)



 Table 13.1. Survey of experimental methods for direct and indirect diffusion studies in solids

Direct methods	Indirect methods	
Tracer diffusion	Mechanical spectroscopy	
plus depth profiling	(after effect, internal friction, Gorski effect)	
Chemical diffusion plus profiling	Magnetic relaxation	
Profiling techniques:	(for ferromagnetc materials)	
- Mechanical and sputter profiling	Nuclear magnetic relaxation (NMR):	
- Secondary ion mass spectrometry (SIMS)	- Line-shape spectroscopy	
- Electron microprobe analysis (EMPA)	- Spin lattice relaxation spectroscopy	
- Auger electron spectroscopy (AES)	- Spin alignment experiments (SAE)	
Spreading resistance profiling (SRP) for semiconductors	<b>Impedance spectroscopy</b> (IS) for ion conductors	
Rutherford backscattering (RBS)	Mössbauer spectroscopy (MBS)	
Nuclear reaction analysis (NRA)	Quasielastic neutron scattering (QENS)	
Field gradient NMR (FG-NMR)	· · · · · · · · · · · · · · · · · · ·	
Pulsed fieldgradient NMR (PFG-NMR)		

...further reading

Neutron scattering X-ray scattering γ-absorption	G. Vogl, B. Sepiol, The Elementary Diffusion Step in Metals Studied by the Interference of Gamma-rays, X-rays and Neutrons, in: Diffusion in Condensed Matter - Methods, Materials, Models, P. Heitjans, J. Kärger (Eds.), Springer, Berlin, 2005, pp. 65-92.
NMR	P. Heitjans, A. Schirmer, S. Indris, NMR and $\beta$ -NMR Studies of Diffusion in Interface-Dominated and Disordered Solids, ibidem, pp. 369-416.
Mössbauer spectroscopy	<ul> <li>G. Sepiol, B. Sepiol, Acta metall. Mater. 42 (1994) 3175-3181; R. Feldwisch,</li> <li>B. Sepiol, G. Vogl, Acta metall. Mater. 43 (1995) 2033-2039.</li> </ul>

General aspects of pulse methods (adsorption and diffusion)

# Pulse methods

- response to *pertubations*  $\rightarrow$  describe mathematically the transient behaviour
- evaluation of rate parameters from response measurements such as mass transfer coefficients, diffusivities, and chemical kinetic constants
- use of: fixed-bed (column) chromatography
  - isotope technique
  - slurry adsorber
  - single-pellet

## Pulse methods



Figure 4.21 Response to a concentration change at the face of a porous pellet: (a) pulse input; (b) step input.

# Signals



...chemical reactors without any reaction may be regarded as linear

## **Frequency Response Method**

# Application background – sorption, diffusion

- Zeolites and microporous solids as *adsorbents* in *gas separation* processes
  - separation factors depend on differences in standard free energies of adsorption of the components
  - when sizes between adsorbates and channels are similar diffusion also influences separation
- Commercial separation units contain pellets + inert binder

→ intercrystalline and intracrystalline diffusion may be important in combination with adsorption Methods for determination of inter- and intracrystalline diffusion

 Measure of uptake rates of an adsorbate molecule from the gas phase into the zeolite pellet

! often spurious diffusion coefficients because of complex interplay with heats of adsorption and bed-depth mass transport effects

- Several methods try to eliminate these problems:
  - Pulsed field gradient NMR
  - Tracer desorption NMR
  - Zero bed-depth chromatography
  - Single-crystal membrane
  - Frequency response
  - > TAP

Provide opportunities to

o determine *rate processes* within porous systems by measuring the pressure response of a closed system to a small fluctuation in its volume

o obtain *kinetic parameters* for chemical reactions and adsorptiondesorption steps on surfaces

o measure *effective diffusivities* within microporous solids

Provide opportunities to

o study *coupled adsorption* and *diffusion* processes with different *characteristic time scales* 

These rate processes *cannot* be adequately *resolved by pulse* or *step transient methods*, by *TPD* or by *elution chromatography*.

o determine *coupling* of adsorption and transport particularly for *high surface area materials* 

(number of exchanging molecules inside > number of exchanging molecules outside)

#### Principle

o FR exploits the *change* in *amplitude* and *phase* produced by a dynamic system that is pertubed periodically around its equilibrium point

o amplitude change (*attenuation*) and phase shift (*lag*) are *directly related* to the *dynamics and capacities* of the processes (sorption and diffusion) governing the return to the equilibrium



Graphical description of the frequency modulation approach

- at each frequency w, the system volume is pertubed sinusoidally with amplitude v (solid line)
- the system pressure responds in a sinusoidal manner with a fluctuation amplitude p (dotted line)
- The fluctuation amplitude p is lower than v and is *delayed* by an *angle*  $\varphi$



Figure 1. Input volume perturbation and output pressure response in frequency response experiments.

#### Extraction of rate parameters

- capacities and dynamics of underlying processes are obtained by *fitting* of the *experimental transfer function* H(v, p, φ) to the theoretical transfer function H(i ω)
- measured quantities  $\nu$ ,  $\rho$ ,  $\varphi$
- transfer function: analytical expression in frequency domain which contains parameters
- magnitude of function measures the ratio of the pressure response to the volume pertubation: |H| = p/v
- quantifies phase lag :  $\varphi = \tan^{-1} [\ldots]$



Figure 1. Input volume perturbation and output pressure response in frequency response experiments.

Experimental apparatus

- Pertubation frequencies between 0.05 10 Hz
- System volume V = 141,91 cm<sup>3</sup> can be modulated by 1,38 % ( $\Delta$ V = 1,96 cm<sup>3</sup>)
- Modulation by metal bellow
- System volume at T = 223-400 K (pretreatment e.g. heating before experiment via resistive heating device)



Figure 2. Schematic diagram of frequency response apparatus.

#### Experimental apparatus

- geometry of system volume minimizes hydrodynamic delays that are not related to dynamics of intraparticle processes
- volume = sample cell, steel bellow and pressure gauge (MKS model 223 BD, 0-130 Pa)
- steel bellow driven by a rotary motor with an acentric cam
- dynamic system pressures of 10<sup>-9</sup> Pa by pump system (turbo+mechanical)



Figure 2. Schematic diagram of frequency response apparatus.

#### Experimental apparatus

- position of bellow is determined by linear variable differential transformer
- Data aquisition: input voltage of motor, phase lags, amplitude attenuations
- Typical sample amount: mesoporous silica spheres (Shell S980B, 204 m<sup>2</sup>/g) ≈ 16 g



Figure 2. Schematic diagram of frequency response apparatus.



RRF and IRF components for coupled diffusion and adsorption processes





Advantages

- o FR allows to apply an *additional degree of freedom* in choosing frequencies to *decouple multiple dynamic processes*
- o FR delivers a *high accuracy* in realizing a *smooth forcing function*
- o FR is able to conduct experiments at mean operating conditions
- o FR anaylsis is relatively easy
- o FR enables to *discriminate* between *different rate-limiting steps*
- o FR weights all parts of response uniformly (no error build up in data analysis)

#### **Evaluation**

o *Bode plot* is a graph of the transfer function of a linear, time-invariant system versus frequency, plotted with a *log-frequency axis*, to show the system's frequency response



Figure 2. Bode plot for fundamental amplitude ratios.



Figure 3. Bode plot for first harmonic amplitude ratios.
# Anomalities in adsorption of hydrocarbons in zeolites

Anomalous adsorption properties can be attributed to entropic or enthalpic effects or combination of both effects

- to energetically heterogeneous adsorption sites
- to strong sorbate-sorbate interactions

How and when to realize such effects

- → like heterogeneity of sites?
  - complex profiles of isosteric heat of adsorption in calorimetry
  - computer simulations of adsorption complexes
- → sorbate-sorbate interactions
  - more to consider for aromatics than for alkanes in zeolites
  - will lead to redistribution and /or reorientation of sorbed molecules

# Anomalities in diffusion of hydrocarbons in zeolites

Anomalous diffusion properties can be also attributed to entropic or enthalpic effects or combination of both effects and depend (next to T, p) on

- characteristics of sorbate
- structure of zeolite channel

How and when to realize such effects

→ characteristic of sorbate (channel)

- bimodal FR spectra for C<sub>1</sub>-C<sub>6</sub> which results from dissipation of heat of adsorption between sorbent and surrounding
- > at low loadings: pure single diffusion
- > at high loadings: effect of heat of adsorption on diffusion
- chain-length dependence of intracrystalline diffusion coefficients
- Saturated cyclic hc diffuse more slowly than their aromatic equivalents

# Anomalities in adsorption and diffusion of hydrocarbons in zeolites



Fig. 9. Chain-length dependence of the self-diffusion coefficients of the *n*-alkanes in silicalite-1 at 303 K derived from the FR technique ( $-\Box$ -) compared with the results measured by PFG NMR [46] at 298 K (-O-), OENS [47] ( $-\Delta$ -) and molecular dynamic calculations [45] ( $-\Delta$ -).

Diffusion coefficients of the o	yclic hydrocarbons in silicalite-1	measured by the FR method
---------------------------------	------------------------------------	---------------------------

Sorbate	T (K)	Silicalite-1 (A)			Silicalite-1 (B)		
		n (m/u.c.)	$D_{01} \times 10^{13} \ (\text{m}^2 \ \text{s}^{-1})^{\text{a}}$	$D_{02} \times 10^{13} (\text{m}^2 \text{s}^{-1})^{\text{b}}$	n (m/u.c.)	$D_{01} \times 10^{13} (\text{m}^2 \text{s}^{-1})^{\text{c}}$	$D_{02} \times 10^{13} \ (\text{m}^2 \text{s}^{-1})^{\text{b}}$
Benzene	273	7.1	_	1.56	5.8	19.4	0.53
	303	_	-	-	5.6	14.6	0.52
	373	1.1	-	2.72	1.1	-	2.58
Toluene	323	_	_	_	4.1	0.35	_
	373	_	-	-	2.0	9.6	0.83
	415	_	-	-	0.5	13.4	_
EB <sup>c</sup>	373	_	_	_	2.9	1.02	_
	415	_	-	-	0.81	7.04	_
p-Xylene	323	4.9	7.4	_	_	_	_
	373	3.9	160	15	_	_	_
	473	_	210	-	_	-	-
CP <sup>d</sup>	253	_	_	_	3.8	14.3	0.22
	273	_	_	_	3.6	_	0.83
	373	_	-	-	0.64	9.4	1.5
CHe	423	_	-	-	_	0.06	-
c-DMCH	398	_	-	-	_	< 0.01	-
t-DMCH	398				_	0.55	_
	448				_	2.9	_

<sup>a</sup> Related to the faster diffusion process for benzene and cyclopentane where two processes exist; for the other sorbates, related to the diffusion process along the straight channel direction.

<sup>b</sup> Related to the slower diffusion process where two processes exist or to the single diffusion process for benzene and cyclopentane; for *p*-xylene and toluene, related to the diffusion process down the sinusoidal channel direction.

<sup>c</sup> Ethylbenzene.

<sup>d</sup> Cyclopentane.

e Cyclohexane.

## TAP – method and modeling

### **Unsteady-state experiments**

- Transient conditions can be achieved by introducing different input signals into the reactor
  - $\rightarrow$  variety of methods
  - variation of residence time  $\boldsymbol{\tau}$
  - variation of temperature T
  - variation of feed composition c, p
  - combination of variation of  $\tau,$  T, c or p
- Changes in composition show clearest deviation from steady-state – therefore used quite often
- Signals may be introduced as step or square wave

#### Steady-state methods

- measure overall performance
- give integrated picture of reaction system
- have minimum reactor residence time of 1 s

#### Transient methods

- give information on individual steps
- operate in millisecond time regime; resolution increase

## TAP – A transient technique

- The key feature which distinguishes it from other pulse experiments is that no carrier gas is used and, gas transport is the result of a pressure gradient.
- At low pulse intensities the total gas pressure is very small, and gas transport occurs via *Knudsen diffusion* only.
- Pulse residence time under vacuum conditions is much shorter than in conventional pulse experiments. Thus a *high time resolution* is achievable.

## **TAP** – Features

- *Extraction of kinetic parameters* differs compared to steady-state and surface science experiments
  - Steady-state: kinetic information is extracted from the transport-kinetics data by experimentally eliminating effects of transport
  - Surface science: gas phase is eliminated
  - In *TAP* pulse experiments *gas transport is not eliminated*. The pulse response data provides in-formation on the transport and kinetic parameters.

## **TAP** – Features

- *TAP pulse* experiments are *state-defining* 
  - Typical pulse contains  $\approx 10^{13} 10^{14}$  molecules

or 10<sup>10</sup> - 10<sup>9</sup> moles

- Example:
  - Sulfated zirconia: 100 m<sup>2</sup>/g with 3 wt.% sulfur  $\rightarrow$  5\*10<sup>18</sup> S/m<sup>2</sup>
  - 1 pulse of *n*-butane: 1\*10<sup>14</sup> molecules
  - per pulse 1/50000 of surface addressed

Simplified schematic of a TAP pulse response experiment



- Injection of a narrow gas pulse into an evacuated microreactor
- Gas pulses travel through the reactor
- Gas molecules (reactant and product) are monitored as a function of time and produce a transient response at the MS

### TAP – System hardware



## TAP – System hardware



- 1) High speed pulse valve
- 2) Pulse valve manifold
- 3) Microreactor
- 4) Mass spectrometer
- 5) Vacuum valve
- 6) Manual flow valve
- 7) Mosfet switch

Characteristic pulse response experimental outputs



## TAP – Multipulse experiment



## TAP – Pump probe experiment



In contrast to traditional kinetic methods, that measure concentrations, the *observable quantity* in TAP pulse response experiments is the *time dependent gas flow* escaping from the outlet of the microreactor.

The outlet flow is measured with a *MS* (QMS) that detects individual components of the flow with great sensitivity.

The *composition* of the flow provides information on the types of *chemical transformation* in the microreactor.

The *time dependence* of the flow contains information on *gas transport* and *kinetics*.

## TAP – Theory

#### Goal

- Interpretation of pulse response data
  - determine typical processes
  - find parameters for these processes
  - develop a model
- Analysis of experiments that provide parameters of diffusion, irreversible adsorption or reaction and reversible reaction

The mathematical framework for the one-zone-model was first published in 1988 (Gleaves).

Basic assumptions of one-zone-model:

- catalyst and inert particle bed is uniform
- no radial gradient of concentration in the bed
- no temperature gradient (axial or radial)
- diffusivity of each gas is constant

#### The gas transport is the result of Knudsen diffusion.

An important *characteristic* of this tranport process is that the *diffusivities* of the individual components of a gas mixture are *independent* of the *pressure* or the *composition* of the mixture.

$$D_{e1} \frac{\sqrt{M_1}}{\sqrt{T_1}} = D_{e2} \frac{\sqrt{M_2}}{\sqrt{T_2}}$$

(1) Diffusion only case -

Mass balance for a non-reacting gas A transported by Knudsen diffusion

$$\varepsilon_{b} \frac{\partial C_{A}}{\partial t} = D_{eA} \frac{\partial^{2} C_{A}}{\partial z^{2}}$$

$$\begin{array}{cccc} C_A & \dots & \text{concentration of gas A (mol/cm^3)} \\ D_{eA} \dots & \text{effective Knudsen diffusivity of} \\ & gas A (cm^2/s) \\ t & \dots & time (s) \\ z & \dots & axial coordinate (cm) \\ \epsilon_b & \dots & fractional voidage of the packed \\ & bed in the reactor \end{array}$$

#### Characteristic fingerprints for response curves



- a) Standard diffusion curve showing key time characteristics and the criterion for Knudsen diffusion
- b) Comparison of standard curve with experimental inert gas curve over inert packed bed

## TAP – Transport + adsorption model

(2)Diffusion + irreversible adsorption

(adsorption is first order in gas concentration)

$$\varepsilon_{b} \frac{\partial C_{A}}{\partial t} = D_{eA} \frac{\partial^{2} C_{A}}{\partial z^{2}} - a_{s} S_{v} (1 - \varepsilon_{b}) k_{a} C_{A}$$

$$\frac{\partial \theta_A}{\partial t} = k_a C_A$$

- surface concentration of active sites a<sub>s</sub> ... (mol/cm<sup>2</sup> of catalyst)
- $k_a \dots S_v \dots$ adsorption rate constant (cm<sup>3</sup> of gas/mol s)
  - surface area of catalyst per volume of catalyst (cm<sup>-1</sup>)
- $\theta_{\mathsf{A}} \dots$ fractional surface coverage of A

Exit flow curve for the diffusion+irreversible adsorption case



Comparison of irreversible adsorption curves with standard diffusion curve (A)  $k_a=0$  (SDC), (B)  $k_a=3$  (C)  $k_a=10$ 

## TAP – Transport and adsorption model

(3) Diffusion + reversible adsorption

Mass balances

- for component A in gas phase

$$\varepsilon_{b} \frac{\partial C_{A}}{\partial t} = D_{eA} \frac{\partial^{2} C_{A}}{\partial z^{2}} - a_{s} S_{v} (1 - \varepsilon_{b}) (k_{a} C_{A} - k_{d} \theta_{A})$$

- for component A on the catalyst surface

$$\frac{\partial \theta_A}{\partial t} = k_a C_A - k_d \theta_A$$

 $k_d$  ... desorption rate constant (s<sup>-1</sup>)

Exit flow curve for the diffusion+reversible adsorption case



Comparison of reversible adsorption curves with standard diffusion curve (A)  $k_a=0$  (SDC), (B)  $k_a=20$ ,  $k_d=20$ , (C)  $k_a=20$ ,  $k_d=5$ 

## Applications

# Sulfated zirconias - catalysts and porous model systems

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# Kinetic modeling

#### Knowledge-Based Approach: Microkinetic Analysis



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## Thank you for your attention!

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